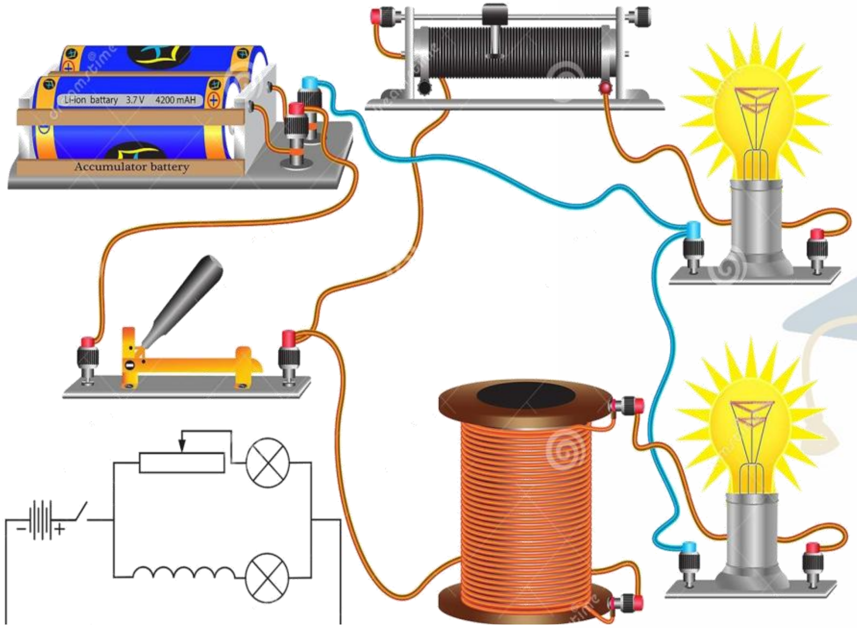
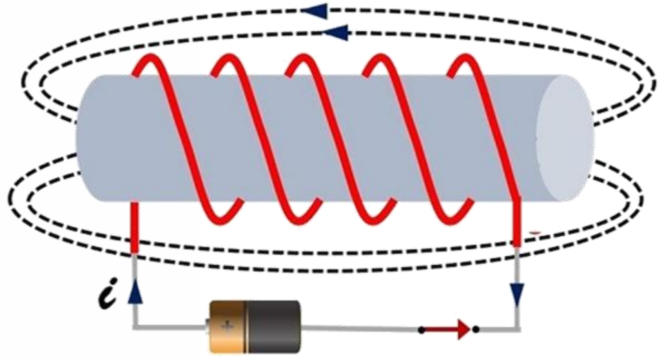


What is Self Induction?

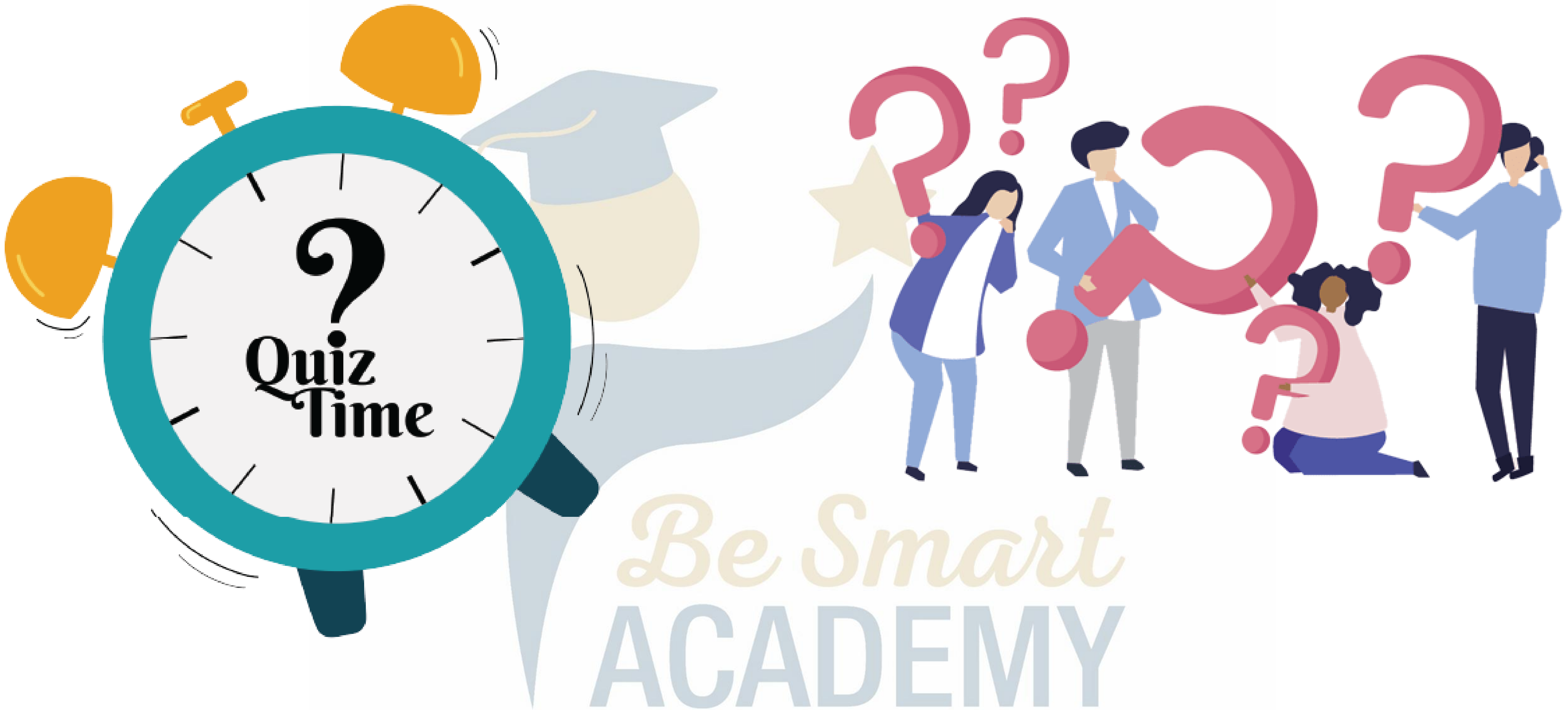


Physics – Grade 12

Unit Two – Electricity

Chapter 9 – Self Induction

Prepared & Presented by: Mr. Mohamad Seif


Quiz Time
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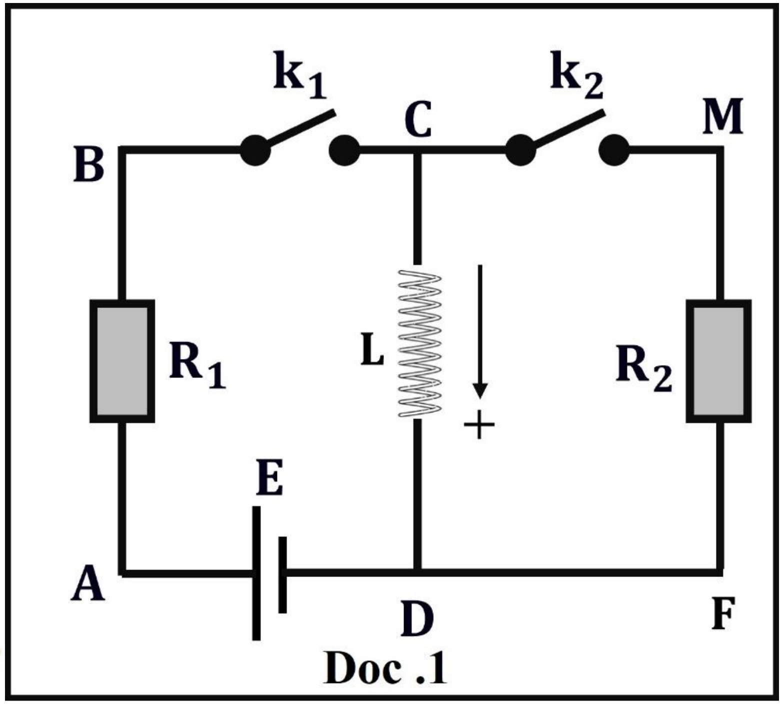
Quiz Self – induction 25 min

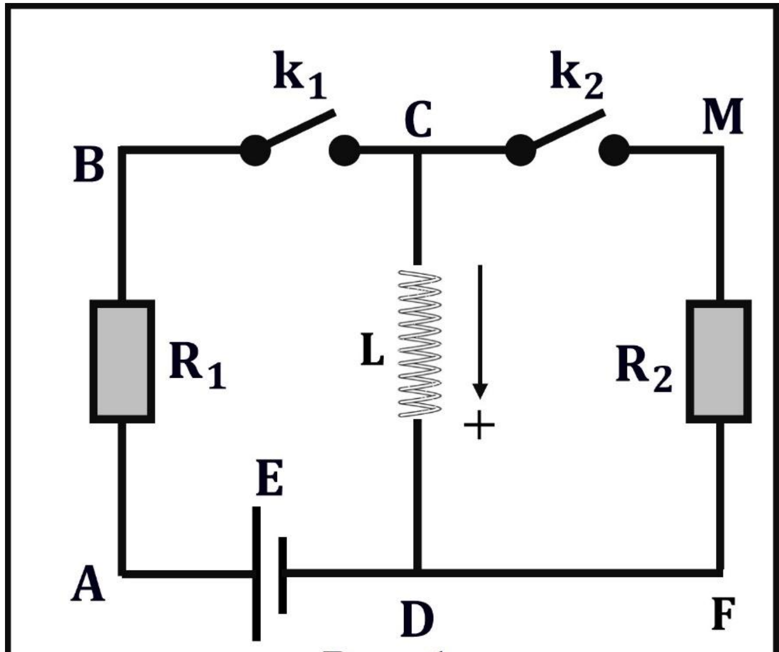
Consider the circuit represented in document 1 that consists of:



- (G) a DC generator of e.m.f $E = 15V$ and of negligible internal resistance.
- (D_1) a resistor of resistance $R_1 = 80\Omega$.
- (D_2) a resistor of resistance $R_2 = 100\Omega$
- (B) a coil of inductance L and of negligible resistance.
- (k_1) and (k_2) are two switches.

The aim of this exercise is to determine the inductance L of the coil

Part A: Growth of the current:



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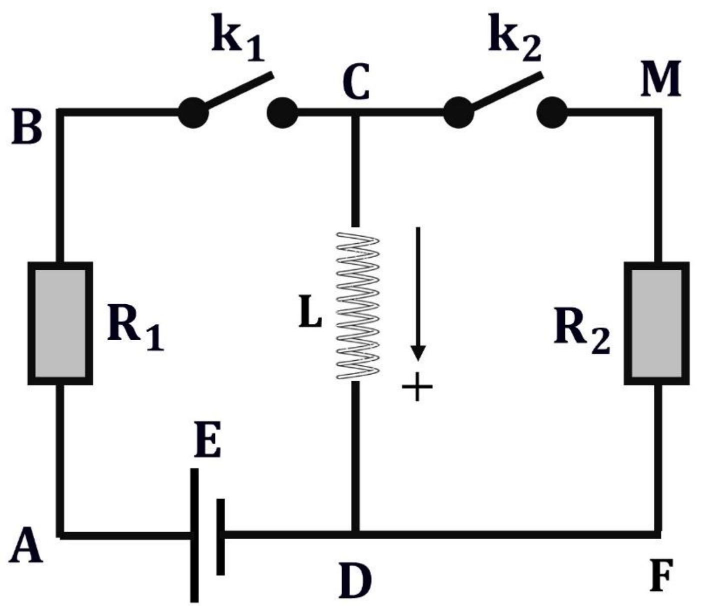
At $t_0 = 0$, we close K_1, and we leave K_2 open:

The circuit carries a current i_1 and the steady state is reached after a certain delay.

1) Name the physical phenomenon responsible for the delay in the growth of the current.

The physical phenomenon responsible of the delay in the growth of the current is called self induction.

2) Explain qualitatively the cause of this delay.



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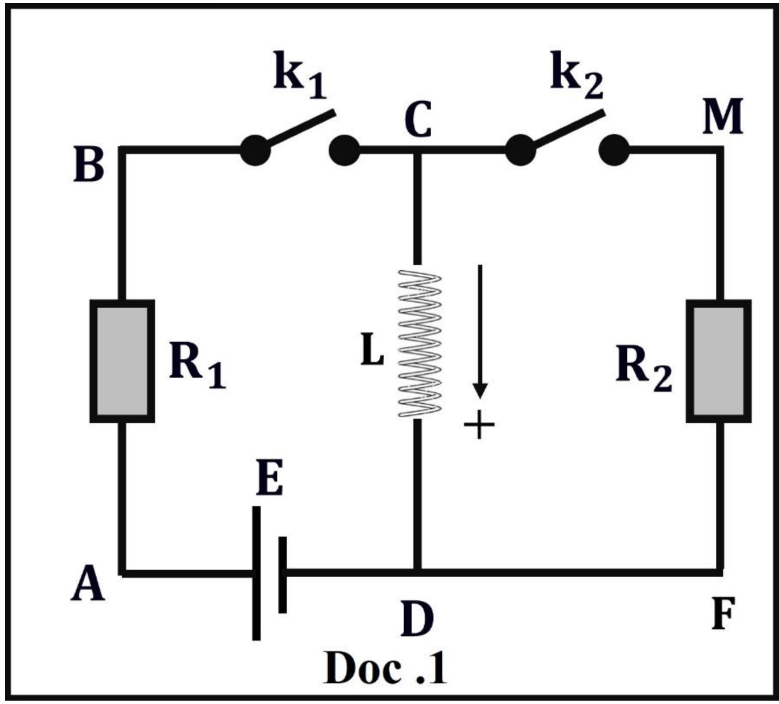
when k_1 is closed and k_2 is opened, the current in the coil starts increasing.

Due to the increase of this current in the coil, the magnetic field and the magnetic flux linked with the coil also increase.

As a result of this process, induced EMF is set up in the coil, such that it opposes the growth of current in the coil .

This delays the current to acquire the maximum value.

3) Derive the differential equation in i_1.



$$u_g = u_{D1} + u_B$$

$$E = ri_1 + L\frac{di_1}{dt} + R_1 i_1$$

$$E = 0 + L\frac{di_1}{dt} + R_1 i_1$$

$$E = R_1 i_1 + L\frac{di_1}{dt}$$

Differential equation in terms of current i_1

4) Verify that $i_1 = \frac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}t}\right)$ is a solution of the preceding differential equation.

$$i = \frac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}t}\right) \Rightarrow \frac{di}{dt} = \frac{E}{R_1} \cdot \frac{R_1}{L} \cdot e^{-\frac{R_1}{L}t}$$

$$\frac{di}{dt} = \frac{E}{L} \cdot e^{-\frac{R_1}{L}t}$$

Substitute i and $\frac{di}{dt}$ in differential equation.

$$E = R_1 i + L\frac{di}{dt}$$

$$E = R_1 \frac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}t}\right) + L.\frac{E}{L}.e^{-\frac{R_1}{L}t}$$

$$E = E\left(1 - e^{-\frac{R_1}{L}t}\right) + E.e^{-\frac{R_1}{L}t}$$

$$E = E - Ee^{-\frac{R_1}{L}t} + E.e^{-\frac{R_1}{L}t}$$

$$E = E$$

Then $i_1 = \frac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}t}\right)$ is a solution of the diff. equation.

5) Determine, in the steady state, the expression of the current I_0 in terms of E and R_1. Calculate its value.

$$i_1 = \frac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}t}\right)$$

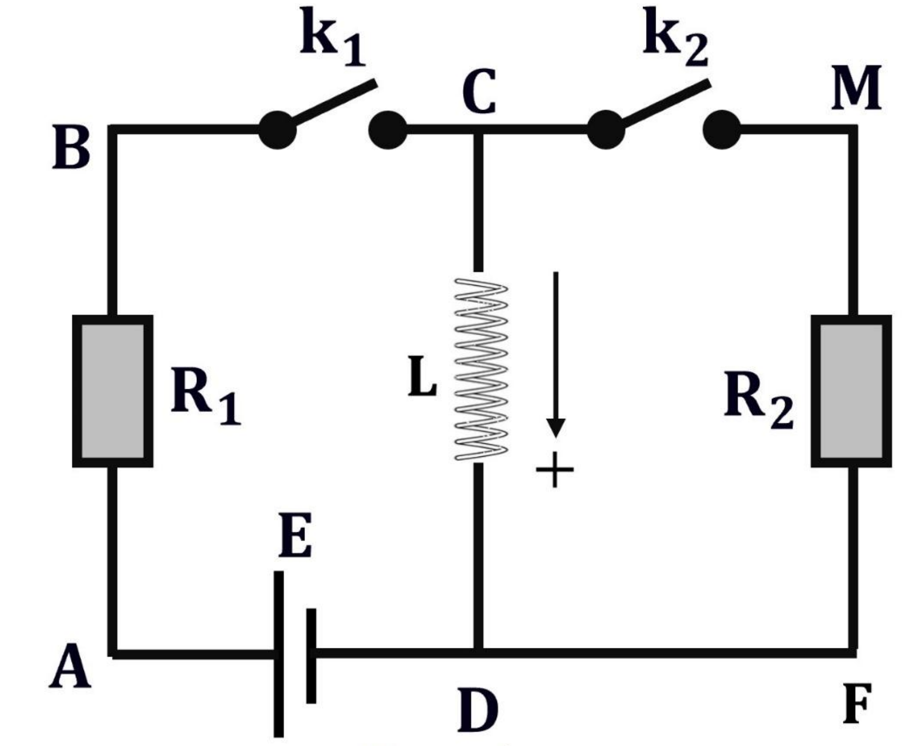
At $t = 5\tau = 5\frac{L}{R_1}$; $i = I_0$

$$I_0 = \frac{E}{R_1}\left(1 - e^{-\frac{R_1}{L}.5\frac{L}{R_1}}\right) \Rightarrow I_0 = \frac{E}{R_1}\left(1 - e^{-5}\right)$$

$$I_0 = 0.99\frac{E}{R_1} \Rightarrow I_0 = \frac{E}{R_1} \Rightarrow I_0 = \frac{15}{80} = 0.1875A$$

Part B: Decay of the current:

At an instant $t_0 = 0$, chosen as a new origin of time, we open k_1 and close k_2 at the same time.



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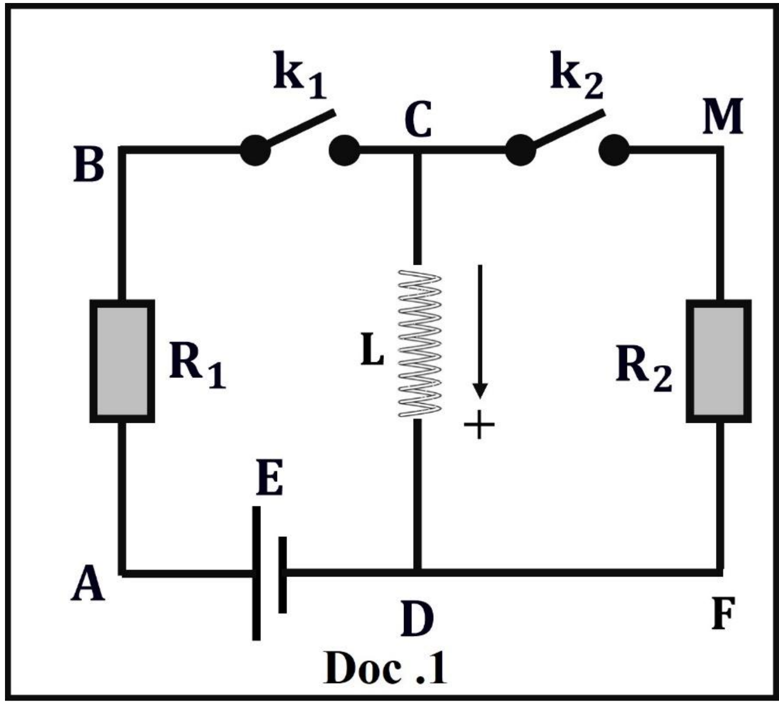
At an instant t, the circuit carries thus a current i_2.

1)Derive the differential equation in i_2.



Apply law of addition of voltages:

$$u_g = u_L + u_{D2}$$

$$0 = ri_2 + L\frac{di_2}{dt} + Ri_2$$

$$0 = R_2 i_2 + L\frac{di_2}{dt}$$

Differential equation in terms of current i_2

2) The solution of this differential equation is of the form $i_2 = \alpha . e^{-\frac{t}{\tau}}$. Show that $\alpha = I_0$ and $\tau = \frac{L}{R_2}$.

$$i_2 = \alpha . e^{-\frac{t}{\tau}} \Rightarrow \frac{di_2}{dt} = -\frac{\alpha}{\tau} . e^{-\frac{t}{\tau}}$$

Substitute i *and* $\frac{di}{dt}$ *in differential equation.*

$$0 = R_2 i_2 + L\frac{di_2}{dt}$$

$$R_2 . \alpha . e^{-\frac{t}{\tau}} + L . \left(-\frac{\alpha}{\tau} . e^{-\frac{t}{\tau}}\right) = 0 \Rightarrow R_2 . \alpha . e^{-\frac{t}{\tau}} + -\frac{\alpha L}{\tau} . e^{-\frac{t}{\tau}} = 0$$

$$\alpha . e^{-\frac{t}{\tau}} \left[R_2 - \frac{L}{\tau}\right] = 0$$

$$\alpha . e^{-\frac{t}{\tau}}\left[R_2 - \frac{L}{\tau}\right] = 0$$

$$R_2 - \frac{L}{\tau} = 0 \Rightarrow R_2 = \frac{L}{\tau} \Rightarrow \tau = \frac{L}{R_2}$$

At $t_0 = 0; i_2 = I_0 = \frac{E}{R_2}$

$$i_2 = \alpha . e^{-\frac{t}{\tau}} \Rightarrow \frac{E}{R_2} = \alpha . e^0 \Rightarrow \alpha = I_0 = \frac{E}{R_2}$$

$$i_2 = \frac{E}{R_2} e^{-\frac{R_2}{L}t}$$

3) **Show that the power dissipated by the resistor is given by:**

$$P = \frac{E^2}{R_2} e^{-2\frac{t}{\tau}}.$$

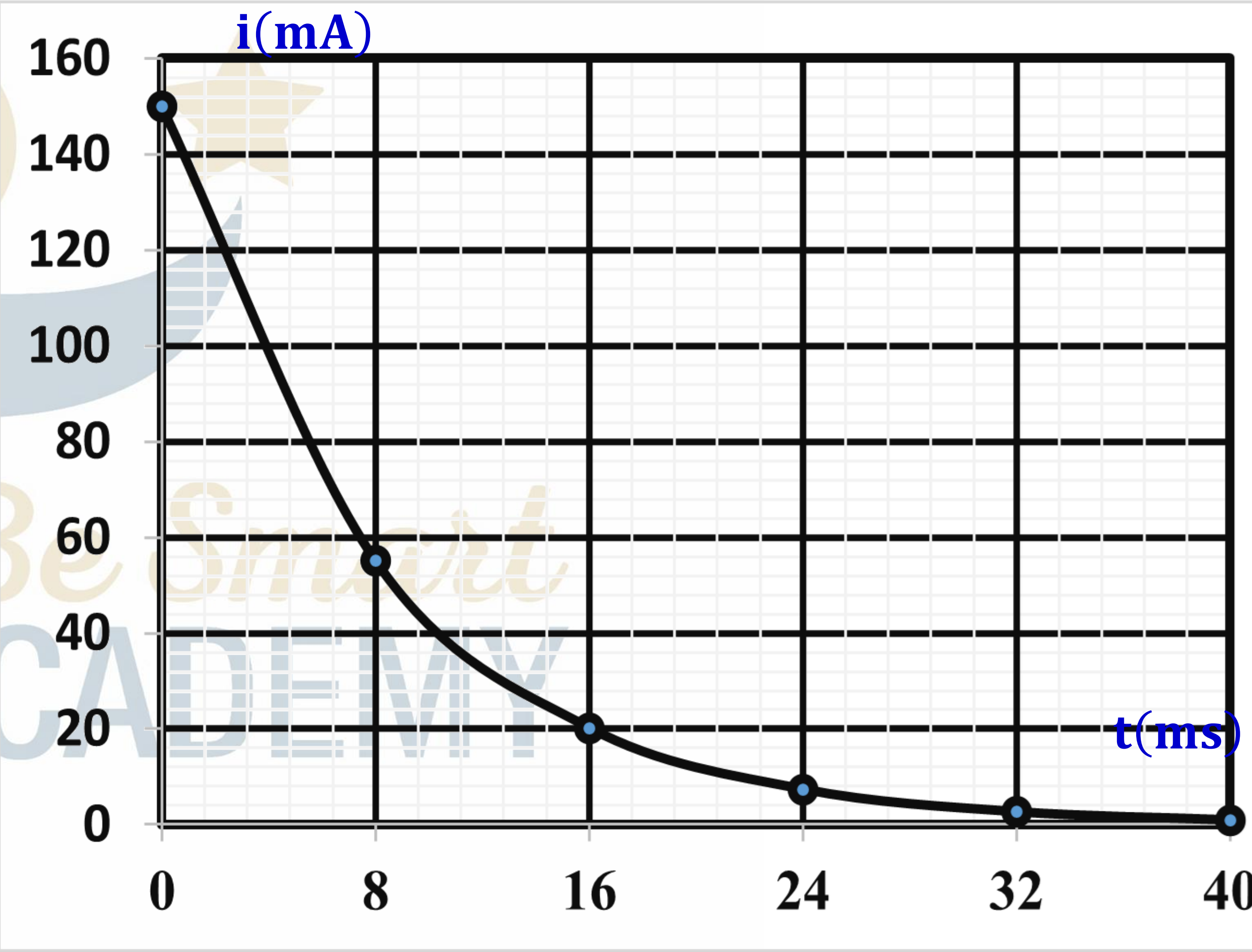
$$P = U \times i_2 \Rightarrow P = (R_2 \times i_2) \times i_2 \Rightarrow P = R_2 \times i_2^2$$

$$P = R_2 \times \left[\frac{E}{R_2} e^{-\frac{t}{\tau}}\right]^2 \Rightarrow P = R_2 . \frac{E^2}{R_2^2} e^{-2\frac{t}{\tau}}$$

$$P = \frac{E^2}{R_2} e^{-2\frac{t}{\tau}}$$

Part C: experimental study:

A convenient setup is used to plot the curve representing the change of the current as a function of time as shown.



1) Referring to the curve, determine the value of the current at $t_0 = 0$

At $t_0 = 0$ $I_0 = 150mA$

2) Determine the time constant τ

At t = τ;

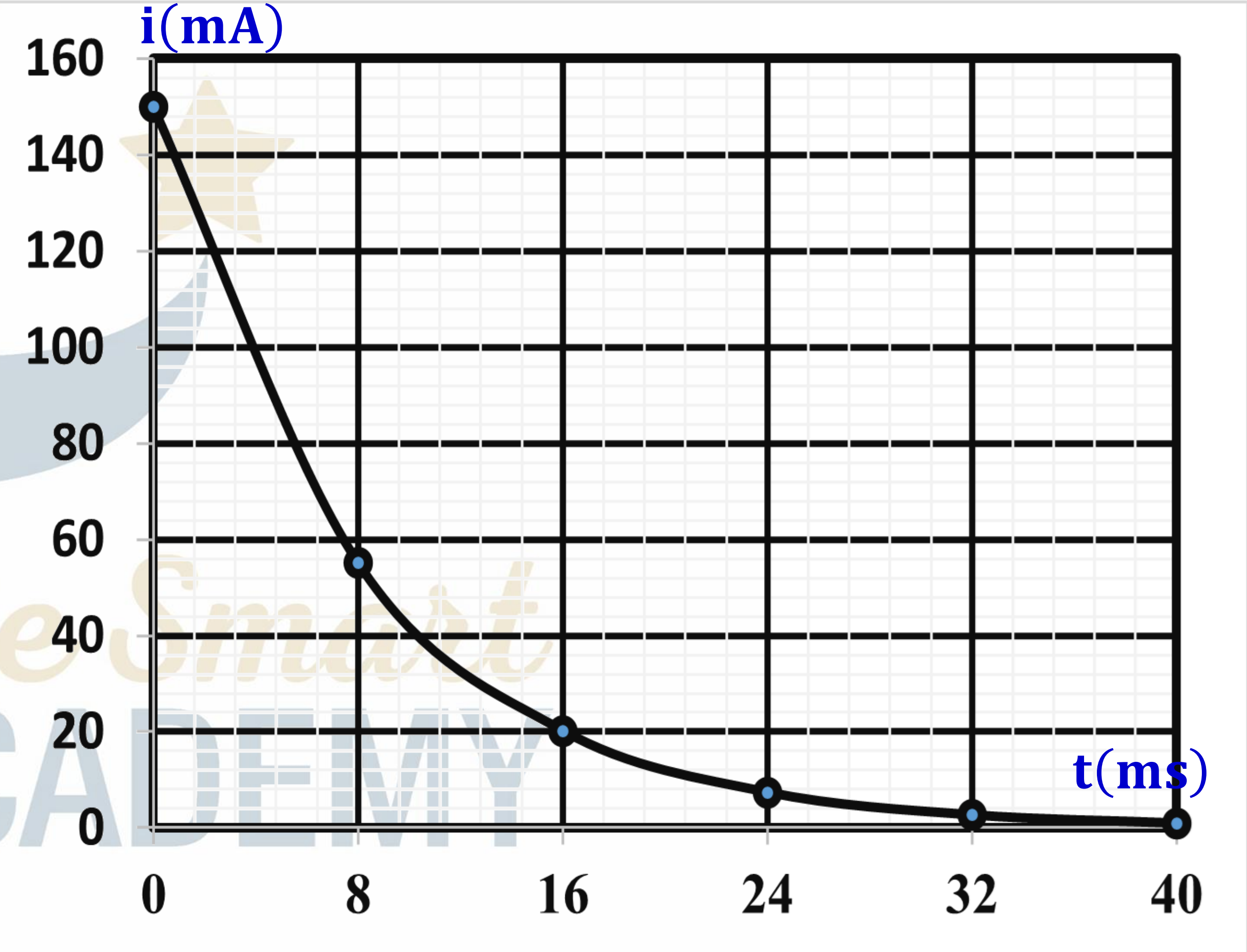
$$i = 0.37 \times I_0$$

$$i = 0.37 \times 150$$

$$i = 55.5mA$$

From graph $i = 55.5mA$ **at** $t = 8ms$

$$\tau = 8ms$$



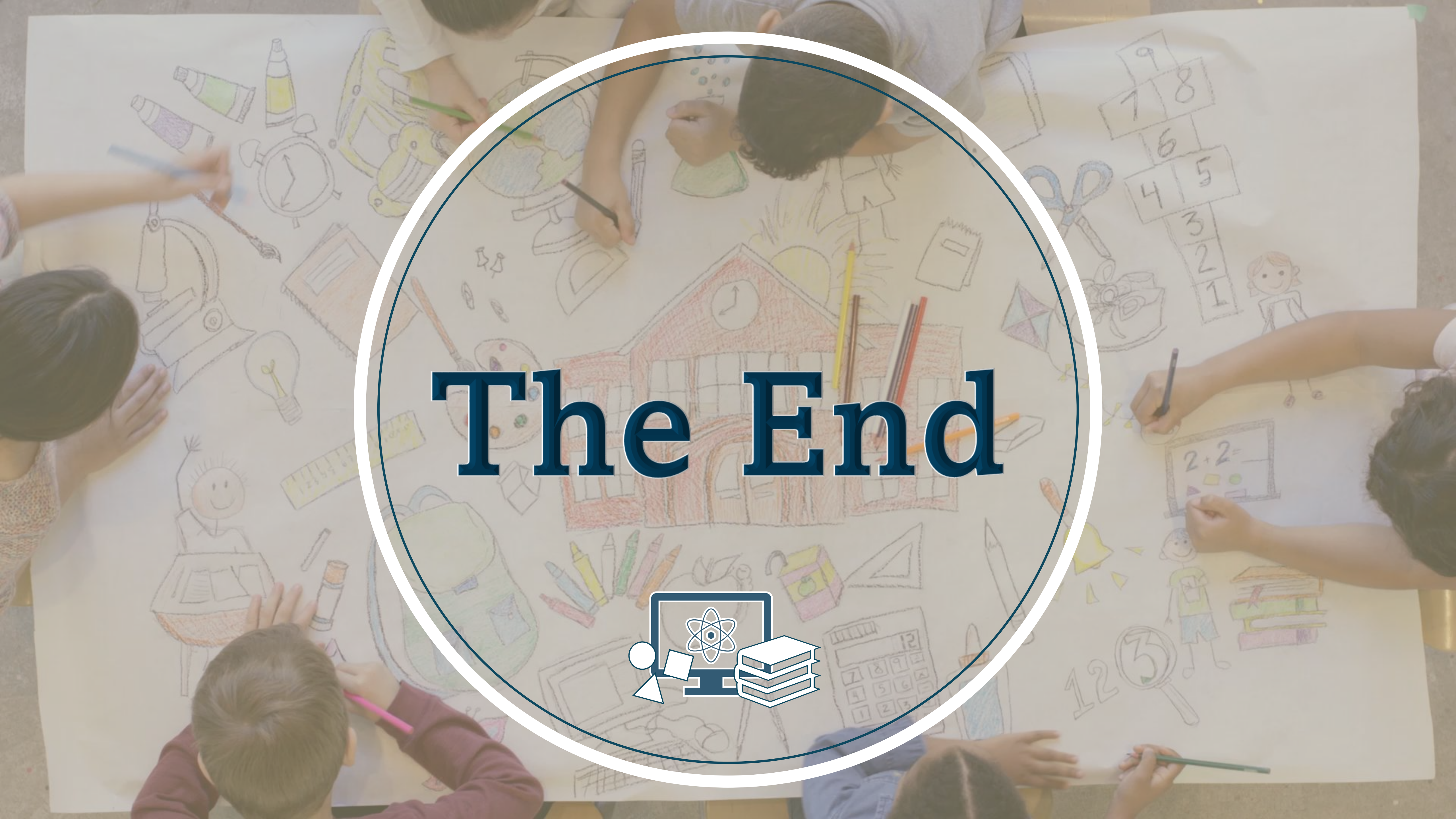
3) Determine the value of the inductance L.

$$\tau = \frac{L}{R_2}$$

$$L = \tau \times R_2$$

$$L = 8 \times 10^{-3} \times 100$$

$$L = 0.8H$$


The End

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